Practice Final Exam

1 Formula Sheet

Newtons Law of Cooling/Heating:

$$\frac{dT}{dt} = k(T - T_1)$$

Useful Trig Identities:

$$\begin{aligned}
\sin^2(x) + \cos^2(x) &= 1 \\
\tan^2(x) + 1 &= \sec^2(x) \\
\sin^2(x) &= \frac{1 - \cos(2x)}{2} \\
\cos^2(x) &= \frac{1 + \cos(2x)}{2} \\
\sin(2x) &= 2\sin(x)\cos(x) \\
\cos(2x) &= \cos^2(x) - \sin^2(x) \\
\sin(\cos^{-1}(x)) &= \sqrt{1 - x^2} \\
\cos(\sin^{-1}(x)) &= \sqrt{1 - x^2} \\
\sec(\tan^{-1}(x)) &= \sqrt{1 + x^2} \\
\tan(\sec^{-1}(x)) &= \begin{cases} \sqrt{x^2 - 1} & x \geq 1 \\ -\sqrt{x^2 - 1} & x \leq -1 \end{cases} \\
\sin(mx)\cos(nx) &= \frac{1}{2}[\sin(m+n)x + \sin(m-n)x] \\
\sin(mx)\sin(nx) &= -\frac{1}{2}[\cos(m+n)x - \cos(m-n)x] \\
\cos(mx)\cos(nx) &= \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]
\end{aligned}$$

Useful Derivatives:

$$\begin{aligned}
D_x \sin^{-1}(x) &= \frac{1}{\sqrt{1 - x^2}} \quad -1 < x < 1 \\
D_x \cos^{-1}(x) &= -\frac{1}{\sqrt{1 - x^2}} \quad -1 < x < 1 \\
D_x \tan^{-1}(x) &= \frac{1}{1 + x^2} \\
D_x \sec^{-1}(x) &= \frac{1}{|x|\sqrt{x^2 - 1}} \quad |x| > 1
\end{aligned}$$

Some Useful Integral Forms:

$$\int \tan u du = -\ln|\cos(u)| + C$$
$$\int \cot(u) du = \ln|\sin(u)| + C$$
$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + C$$
$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + C$$
$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + C$$

Series Formulas:

Taylor's Formula

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)(x-a)^n}{n!}$$

Hyperbolic Trig Definitions:

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

Area of a polar graph:

$$A = \frac{1}{2}\int_a^b (f(\theta))^2 d\theta$$

Slope of the tangent line at θ:

$$Slope = \frac{f(\theta)\cos(\theta) - f'(\theta)\sin(\theta)}{-f(\theta)\sin(\theta) + f'(\theta)\cos(\theta)}$$

Note: The practice final is far from comprehensive but should give you an idea of the types of questions that could appear on the final exam

2 True/False

1. Review the True/False questions from practice midterms 1-3 and midterms 1-3.
2. A polar coordinate function can fail the vertical line test and still be a function.
3. The origin in polar coordinates could be defined as the ordered pair $(0, 2\pi - 3)$ in polar coordinates.
4. A polar coordinate function can have vertical tangent lines.
5. There are infinitely many ways to write the same coordinate location in polar coordinates.

3 Free Response

Calculate the Following Derivatives:

1. f'(x) if $f(x) = \sinh(\cos(x^2))$

2. $\frac{d}{dx}\left[3^{\tan(x)}\right]$

Solve the following differential equation:

3. $\dfrac{dy}{dx} = -2y,\ y(0) = 4$

Evaluate the following integrals:

5. $\int x\sqrt{x-4}dx$

6. $\int\limits_{0}^{2} \frac{dx}{(x-1)^2}$

7. $\int \tan^3(x) dx$

8. $\int\limits_{0}^{\infty} x^2 e^{-x} dx$

Evaluate the following limits:

9. $\lim_{x \to \infty} \frac{\ln(x)}{e^x}$

10. $\lim_{x \to 0^+} (x+1)^{\cot(x)}$

Determine whether the following series converge/diverge (absolute or conditional convergence when applicable):

11. $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^2}{n!}$

12. $$\sum_{n=1}^{\infty} \frac{(n+1)(-1)^n}{n^2+3n}$$

13. Find the 4^{th} degree Taylor polynomial for $f(x) = x^2 + \ln x$ about the point x=e

14. For the polar coordinate equation of a limacon $r = 1 - 2\sin\theta$ sketch the graph of the equation.

Then find the area enclosed by the graph.

Finally find the slope of the line tangent to the graph at $\theta = \frac{\pi}{2}$